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Canonical ADM Tetrad Gravity: from Metrological Inertial Gauge Variables to Dynamical Tidal Dirac Observables

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Abstract

In this updated review of canonical ADM tetrad gravity in a family of globally hyperbolic asymptotically Minkowskian space-times without super-translations I show which is the status of the art in the search of a canonical basis adapted to the first class Dirac constraints and of the Dirac observables of general relativity (GR) describing the tidal degrees of freedom of the gravitational field. In these space-times the asymptotic ADM Poincaré group replaces the Poincaré group of particle physics, there is a York canonical basis diagonalizing the York-Lichnerowicz approach and a post-Minkowskian linearization is possible with the associated description of gravitational waves in the family of non-harmonic 3-orthogonal Schwinger time gauges.

Moreover I show that every fixation of the inertial gauge variables (i.e. the choice of a non-inertial frame) of every generally covariant formulation of GR is equivalent to a set of conventions for the metrology of the space-time (like the GPS ones near the Earth): for instance the freedom in clock synchronization is described by the inertial gauge variable York time (the trace of the extrinsic curvature of the instantaneous 3-spaces). This inertial gauge freedom and the non-Euclidean nature of the instantaneous 3-spaces required by the equivalence principle are connected with the dark side of the universe and could explain the presence of dark matter or at least part of it by means of the adoption of suitable metrical conventions for the ICRS celestial reference system. Also some comments on a canonical quantization of GR coherent with this viewpoint are done.

I. INTRODUCTION

The understanding of gravity and of its quantization is one of the most important challenges of contemporary theoretical physics. At the classical level inside the Solar System Einstein GR remains a satisfactory description of all the aspects of gravity [1]. Instead at the astrophysical and cosmological levels the dominance of dark matter and dark energy with respect to ordinary matter has induced the introduction of many possible alternatives to Einstein GR, like for instance $f(R)$ theories (see for instance Refs.[2]). Most of these theories are generally covariant like Einstein GR.

In this paper I make a review of recent developments in a well defined approach to classical canonical tetrad gravity and of their implications for relativistic metrology in astrophysics. This is an upgrading of the old reviews [3], which contained also a survey of classical relativistic mechanics in non-inertial frames of Minkowski space-time in presence of an electro-magnetic field. For an independent review on classical canonical gravity see the first chapter of Ref.[4].

In a series of papers [5–8] I looked to the existing Hamiltonian formulations of metric and tetrad gravity ¹ by taking into account all the aspects of Dirac theory of constraints [10, 11].

The use of Hamiltonian methods restricts the class of Einstein space-times to the *globally hyperbolic* ones, in which there is a global notion of a mathematical time parameter. The space-times must also be *topologically trivial*. At this level there are two classes of physically inequivalent space-times with a completely different dynamical interpretation:

A) *Spatially compact space-times without boundary* - In them the canonical Hamiltonian is zero and the Dirac Hamiltonian is a linear combination of first class constraints. This fact gives rise to a *frozen picture* without a global evolution (the Dirac Hamiltonian generates only Hamiltonian gauge transformations; in the abstract reduced phase space, quotient with respect to such gauge transformations, the reduced Hamiltonian is zero). This class of space-times fits well with Machian ideas (no boundary conditions), with interpretations in which there is no physical time (see for instance Ref.[12]) and is used in loop quantum gravity.

B) *Asymptotically flat space-times* - In them we have the asymptotic symmetries of the SPI group [13] (direction-dependent asymptotic Killing symmetries). If we restrict this class of space-times to those *not containing super-translations* [14], the SPI group reduces to the *asymptotic ADM Poincaré group* ²: these space-times are asymptotically Minkowskian ³ and in the limit of vanishing Newton constant ($G = 0$) the ADM Poincaré group becomes the special relativistic Poincaré group of the matter present in the space-time (this is an important condition for the inclusion of particle physics, whose properties are all connected with the representations of this group in the inertial frames of Minkowski space-time, into GR). In this restricted class the canonical Hamiltonian is the ADM energy [5], so that there is no frozen picture (in the reduced phase space there is a non-zero reduced Hamiltonian). In absence of matter a sub-class of these space-times is the (singularity-free) family of

¹ Tetrad gravity is needed for the description of fermion fields: its formulation was introduced for the first time in Ref.[9].

² For recent reviews on this group see Refs.[15–17].

³ This class of space-times admits ortho-normal tetrads and a spinor structure [18].

Chrstodoulou-Klainermann solutions of Einstein equations [19] (they are near to Minkowski space-time in a norm sense and contain gravitational waves).

Since the equivalence principle forbids the existence of global inertial frames, we had to define *global non-inertial frames* (with instantaneous non-Euclidean 3-spaces with synchronized clocks) and radar 4-coordinates [20] in this class of space-times by adapting to GR the theory of global non-inertial frames in Minkowski space-time developed in Ref. [21, 22].

In Refs.[24?] there is a systematic study of canonical ADM tetrad gravity (it derives from ADM gravity [25] if in its Lagrangian the 4-metric is decomposed in terms of cotetrads) in *globally hyperbolic, asymptotically Minkowskian space-times without super-translations* with electrically charged positive-energy scalar particles plus the electro-magnetic field as matter.

In this formulation all the 14 constraints are first class ⁴ and I tried to find Shanmugadhasan canonical transformations [26–28] to new canonical bases in which Abelianized forms of many constraints are new momenta (canonical bases adapted to as many as possible constraints). Due to the non-linearity of the super-Hamiltonian and super-momentum constraints, their solution is not known and we are still unable to find their Abelianization and a canonical basis adapted to all the first class constraints of GR.

As a consequence I could only find a York canonical basis [?] adapted to 10 first class constraints and diagonalizing the York-Lichnerowicz approach [29–32]. In this basis one can identify which are: a) the four quantities to be determined by the super-Hamiltonian and super-momentum constraints; b) the gauge variables of GR describing inertial effects (one of them is the York time, i.e. the trace of the extrinsic curvature of the instantaneous 3-spaces); c) the two pairs of dynamical variables describing the tidal effects of GR (the two polarizations of gravitational waves in the linearized theory). Do to the use of radar 4-coordinates *all these quantities are 4-scalars with respect to the ordinary world 4-coordinates*.

Instead the Dirac observables (DO) (gauge invariant under the Hamiltonian gauge transformations generated by all the first class constraints) of the gravitational field are not known: we have only statements about their existence [33–35]. They would be the two pairs of tidal variables in a Shanmugadhasan canonical basis adapted to all the 14 first class constraints. In our approach they would be 4-scalars like the observables of the Hilbert-Einstein action, whose gauge group are the 4-diffeomorphisms of the space-time. See Ref.[36–38] for what is known on the connection between 4-diffeomorphisms and the Hamiltonian gauge group: only on the space of solutions of Einstein equations there is an overlap of the two notions of observable.

Moreover it is possible [39] to find the Hamiltonian expression of the Riemann and Weyl curvature tensors and to give the Hamiltonian formulation of the Newman-Penrose formalism [40].

An extremely important (till now unnoticed) point is that the fixation of the gauge freedom of GR (and of every generally covariant theory of gravity), i.e. the choice of the non-inertial frame and of its 4-coordinates, is nothing else that *the establishment of conventions*

⁴ I consider only space-times without Killing symmetries, because otherwise there would be extra (either first or second class) constraints, Hamiltonian counterpart of the Killing equations. Also the eigenvalues of the 3-metric of the instantaneous 3-spaces must be different: again the degenerate cases can be described by adding the equality of the eigenvalues as extra constraints.

for *relativistic metrology*, an operation done from atomic physicists, NASA engineers and astronomers [41, 43?, 44]. As shown in the third paper of Refs.[24] the inertial gauge freedom in the York time is connected with the existence of dark matter at the cosmological level [45]. It is possible that at least part of dark matter is a relativistic inertial effect eliminable with a suitable convention for the ICRS celestial reference system. Moreover the York time is also connected with dark energy.

After these cosmological considerations I will end with some comments on the quantization of canonical gravity: it is argued that only the final 4-scalar DO's describing the tidal effects should be quantized but not the inertial gauge variables describing the freedom in non-inertial frames.

In Section II I discuss non-inertial frames and radar 4-coordinates in asymptotically Minkowskian space-times. In Section III I define canonical ADM tetrad gravity and its York canonical basis is introduced in Section IV, where there is also a description of the Hamilton equations and of the inertial gauge variable York time. In Section V I introduce the family of non-harmonic 3-orthogonal Schwinger time gauges, I define a Hamiltonian Post-Minkowskian (HPM) linearization and I describe the resulting gravitational waves (GW). In Section VI I study the HPM equations of motion for particles and their Post-Newtonian (PN) expansion, showing that the inertial gauge variable York time allows to interpret (at least part of) astrophysical dark matter as a relativistic inertial effect. In Section VII I discuss relativistic metrology, whose conventions are gauge fixings for the inertial gauge variables of every generally covariant description of GR identifying global non-inertial frames and their 4-coordinates in the space-time.

Then there are some Conclusions containing a sketch of the open problems to be investigated with the described approach.

II. NON-INERTIAL FRAMES IN ASYMPTOTICALLY MINKOWSKIAN SPACE-TIMES AND THE GRAVITATIONAL FIELD

Let us consider globally hyperbolic, topologically trivial, asymptotically Minkowskian space-times without super-translations. In them one can define global non-inertial frames with the same methodology introduced in special relativity (SR) [21].

Assume that the world-line $x^\mu(\tau)$ of an arbitrary time-like observer ⁵ carrying a standard atomic clock is given: τ is an arbitrary monotonically increasing function of the proper time of this clock. Then one gives an admissible 3+1 splitting of the asymptotically flat space-time, namely a nice foliation with space-like instantaneous 3-spaces Σ_τ . It is the mathematical idealization of a protocol for clock synchronization: all the clocks in the points of Σ_τ sign the same time of the atomic clock of the observer ⁶. The observer and the

⁵ An observer, or better a mathematical observer, is an idealization of a measuring apparatus containing an atomic clock and defining, by means of gyroscopes, a set of spatial axes (and then a, maybe orthonormal, tetrad with a convention for its transport) in each point of the world-line.

⁶ It is the non-factual idealization (generalizing the existing protocols for building coordinate systems inside the future light-cone of a time-like observer) required by the Cauchy problem: without it we cannot use the existence and unicity theorem for the solutions of partial differential equations to predict the future

foliation define a global non-inertial reference frame after a choice of 4-coordinates. On each 3-space Σ_τ one chooses curvilinear 3-coordinates σ^r having the observer as origin.

The quantities $\sigma^A = (\tau; \sigma^r)$ are the 4-scalar and observer-dependent *radar 4-coordinates*, first introduced by Bondi [20].

If $x^\mu \mapsto \sigma^A(x)$ is the coordinate transformation from world 4-coordinates x^μ having the observer as origin to radar 4-coordinates, its inverse $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the *embedding* functions $z^\mu(\tau, \sigma^r)$ describing the 3-spaces Σ_τ as embedded 3-manifolds into the asymptotically flat space-time. Let $z_A^\mu(\tau, \sigma^u) = \partial z^\mu(\tau, \sigma^u) / \partial \sigma^A$ denote the gradients of the embedding functions with respect to the radar 4-coordinates. The space-like 4-vectors $z_r^\mu(\tau, \sigma^u)$ are tangent to Σ_τ , so that the unit time-like normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu_{\alpha\beta\gamma} [z_1^\alpha z_2^\beta z_3^\gamma](\tau, \sigma^u)$ ($\epsilon_{\mu\alpha\beta\gamma}$ is the Levi-Civita tensor). Instead $z_\tau^\mu(\tau, \sigma^u)$ is a time-like 4-vector skew with respect to the 3-spaces leaves of the foliation ⁷.

In GR the dynamical fields are the components ${}^4g_{\mu\nu}(x)$ of the 4-metric ⁸ and not the embeddings $x^\mu = z^\mu(\tau, \sigma^r)$ defining the admissible 3+1 splittings of space-time like in the parametrized Minkowski theories of SR [3, 21, 22]. Now the gradients $z_A^\mu(\tau, \sigma^r)$ of the embeddings give the transition coefficients from radar to world 4-coordinates, so that the components ${}^4g_{AB}(\tau, \sigma^r) = z_A^\mu(\tau, \sigma^r) z_B^\nu(\tau, \sigma^r) {}^4g_{\mu\nu}(z(\tau, \sigma^r))$ of the 4-metric will be the dynamical fields in the ADM action [25]. Let us remark that *the ten quantities ${}^4g_{AB}(\tau, \sigma^r)$ are 4-scalars of the space-time due to the use of the 4-scalar radar 4-coordinates*. The same happens for *all the components of "radar tensors" (i.e. tensors expressed in radar 4-coordinates): they are 4-scalars of the space-time*.

The 4-metric ${}^4g_{AB}$ has signature $\epsilon(+---)$ with $\epsilon = \pm$ (the particle physics, $\epsilon = +$, and general relativity, $\epsilon = -$, conventions). Flat indices (α) , $\alpha = o, a$ ($a = 1, 2, 3$), are raised and lowered by the flat Minkowski metric ${}^4\eta_{(\alpha)(\beta)} = \epsilon(+---)$. We define ${}^4\eta_{(a)(b)} = -\epsilon \delta_{(a)(b)}$ with a positive-definite Euclidean 3-metric $\delta_{(a)(b)}$. From now on we shall denote the curvilinear 3-coordinates σ^r with the notation $\vec{\sigma}$ for the sake of simplicity. Usually the convention of sum over repeated indices is used, except when there are too many summations. The symbol \approx means Dirac weak equality, while the symbol $\stackrel{\circ}{=}$ means evaluated by using the equations of motion.

We shall work with the radar tetrads ${}^4E_{(\alpha)}^A(\tau, \vec{\sigma})$ and the radar cotetrads ${}^4E_A^{(\alpha)}(\tau, \vec{\sigma})$. The original tetrads are ${}^4E_{(\alpha)}^\mu(\tau, \vec{\sigma}) = z_A^\mu(\tau, \vec{\sigma}) {}^4E_{(\alpha)}^A(\tau, \vec{\sigma})$.

Since the world-line of the time-like observer can be chosen as the origin of a set of the spatial world coordinates, i.e. $x^\mu(\tau) = (x^o(\tau); 0)$, it turns out that with this choice the

⁷ In SR, see Refs. [21, 22], one has $z_\tau^\mu(\tau, \sigma^r) = [N l^\mu + N^r z_r^\mu](\tau, \sigma^r)$ with $N(\tau, \sigma^r) = \epsilon [z_\tau^\mu l_\mu](\tau, \sigma^r) = 1 + n(\tau, \sigma^r) > 0$ and $N_r(\tau, \sigma^r) = -\epsilon [z_\tau^\mu \eta_{\mu\nu} z_r^\nu](\tau, \sigma^r)$ being the lapse and shift functions respectively of the global non-inertial frame of Minkowski space-time so defined. This decomposition holds also in GR.

⁸ Let us remark that the ten dynamical fields ${}^4g_{\mu\nu}(x)$ are not only a (pre)potential for the gravitational field (like the electro-magnetic and Yang-Mills fields are the potentials for electro-magnetic and non-Abelian forces) but also determines the *chrono-geometrical structure of space-time* through the line element $ds^2 = {}^4g_{\mu\nu} dx^\mu dx^\nu$. Therefore the 4-metric teaches relativistic causality to the other fields: it says to massless particles like photons and gluons which are the allowed world-lines in each point of space-time. This basic property is lost in every quantum field theory approach to gravity with a fixed background 4-metric.

space-like surfaces of constant coordinate time $x^o(\tau) = \text{const.}$ coincide with the dynamical instantaneous 3-spaces Σ_τ with $\tau = \text{const.}$. By using asymptotic flat tetrads $\epsilon_A^\mu = \delta_o^\mu \delta_A^\tau + \delta_i^\mu \delta_A^i$ (with ϵ_μ^A denoting the inverse flat cotetrads) and by choosing a coordinate world time $x^o(\tau) = x_o^o + \epsilon_\tau^o \tau = x_o^o + \tau$, one gets the following preferred embedding corresponding to these given world 4-coordinates $x^\mu = z^\mu(\tau, \vec{\sigma}) = x^\mu(\tau) + \epsilon_\tau^\mu \sigma^r = \delta_o^\mu x_o^o + \epsilon_A^\mu \sigma^A$. This choice implies $z_A^\mu(\tau, \vec{\sigma}) = \epsilon_A^\mu$ and ${}^4g_{\mu\nu}(x = z(\tau, \vec{\sigma})) = \epsilon_\mu^A \epsilon_\nu^B {}^4g_{AB}(\tau, \vec{\sigma})$.

As shown in Ref.[46], the dynamical nature of space-time implies that each solution (i.e. an Einstein 4-geometry) of Einstein's equations (or of the associated ADM Hamilton equations) dynamically selects a preferred family of 3+1 splitting of the space-time, namely in GR the instantaneous 3-spaces (and therefore the associated clock synchronization convention) are dynamically determined modulo only one inertial gauge function. As we will show, in the York canonical basis this function is the *York time*, namely the trace of the extrinsic curvature of the 3-space. While in SR the gauge freedom in clock synchronization depends on four basic gauge functions, the embeddings $z^\mu(\tau, \sigma^r)$, and both the 4-metric and the whole extrinsic curvature tensor are derived inertial potentials, in GR the extrinsic curvature tensor of the 3-spaces is a mixture of dynamical (tidal) pieces and inertial gauge variables playing the role of inertial potentials (but only the York time is a freedom in the choice of the shape of the 3-spaces as 3-sub-manifolds of the space-time).

III. CANONICAL ADM TETRAD GRAVITY

To define the canonical formalism the Einstein-Hilbert action for metric gravity must be replaced with the ADM action (the two actions differ for a surface term at spatial infinity). In the chosen class of space-times the ten strong ADM Poincaré generators P_{ADM}^A, J_{ADM}^{AB} (they are fluxes through a 2-surface at spatial infinity) are given as boundary conditions at spatial infinity. As shown in Ref.[5], the Legendre transform and the definition of a consistent canonical Hamiltonian require the introduction of the DeWitt surface term at spatial infinity: the final canonical Hamiltonian turns out to be the *strong* ADM energy (a flux through a 2-surface at spatial infinity), which is equal to the *weak* ADM energy (expressed as a volume integral over the 3-space) plus constraints. Therefore there is not a frozen picture but an evolution generated by a Dirac Hamiltonian equal to the weak ADM energy plus a linear combination of the first class constraints. Also the other strong ADM Poincaré generators are replaced by their weakly equivalent weak form $\hat{P}_{ADM}^A, \hat{J}_{ADM}^{AB}$.

In Ref.[5] it is also shown that the boundary conditions on the 4-metric required by the absence of super-translations imply that the only admissible 3+1 splittings of space-time (i.e. the allowed global non-inertial frames) are the *non-inertial rest frames*: their 3-spaces are asymptotically orthogonal to the weak ADM 4-momentum. Therefore we get $\hat{P}_{ADM}^r \approx 0$ as the rest-frame condition of the 3-universe with a mass and a rest spin fixed by the boundary conditions. Like in SR the 3-universe can be visualized as a decoupled non-covariant (non-measurable) external relativistic center of mass plus an internal non-inertial rest-frame 3-space containing only relative variables (see the first paper in Ref.[24]).

In tetrad gravity the 4-metric is decomposed in terms of cotetrads, ${}^4g_{AB} = E_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} E_B^{(\beta)}$ and the ADM action, now a functional of the 16 fields $E_A^{(\alpha)}(\tau, \sigma^r)$, is taken as the action for ADM tetrad gravity. The diffeomorphism group (the gauge group of GR) is enlarged with the $O(3,1)$ gauge group of the Newman-Penrose approach [40] (the extra

gauge freedom acting on the tetrads in the tangent space of each point of space-time and reducing from 16 to 10 the number of independent fields like in metric gravity). This leads to an interpretation of gravity based on a congruence of time-like observers endowed with ortho-normal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer's gyroscopes. This framework was developed in Refs.[6, 7].

In this framework the configuration variables are cotetrads, which are connected to cote-trads adapted to the 3+1 splitting of space-time (so that the adapted time-like tetrad is the unit normal to the 3-space Σ_τ) by standard Wigner boosts for time-like vectors ⁹ of parameters $\varphi_{(a)}(\tau, \sigma^r)$: ${}^4E_A^{(\alpha)} = L^{(\alpha)}_{(\beta)}(\varphi_{(a)}) {}^4\overset{\circ}{E}_A^{(\beta)}$. The adapted tetrads and cotetrads have the expression

$$\begin{aligned} {}^4\overset{\circ}{E}_{(o)}^A &= \frac{1}{1+n} (1; -\sum_a n_{(a)} {}^3e_{(a)}^r) = l^A, & {}^4\overset{\circ}{E}_{(a)}^A &= (0; {}^3e_{(a)}^r), \\ {}^4\overset{\circ}{E}_A^{(o)} &= (1+n) (1; \vec{0}) = \epsilon l_A, & {}^4\overset{\circ}{E}_A^{(a)} &= (n_{(a)}; {}^3e_{(a)r}), \end{aligned} \quad (3.1)$$

where ${}^3e_{(a)}^r$ and ${}^3e_{(a)r}$ are triads and cotriads on Σ_τ and $n_{(a)} = n_r {}^3e_{(a)}^r = n^r {}^3e_{(a)r}$ ¹⁰ are adapted shift functions. In Eqs.(3.1) $N(\tau, \vec{\sigma}) = 1 + n(\tau, \vec{\sigma}) > 0$, with $n(\tau, \vec{\sigma})$ vanishing at spatial infinity (absence of super-translations), so that $N(\tau, \vec{\sigma}) d\tau$ is positive from Σ_τ to $\Sigma_{\tau+d\tau}$, is the lapse function; $N^r(\tau, \vec{\sigma}) = n^r(\tau, \vec{\sigma})$, vanishing at spatial infinity (absence of super-translations), are the shift functions.

The adapted tetrads ${}^4\overset{\circ}{E}_{(a)}^A$ are defined modulo $\text{SO}(3)$ rotations ${}^4\overset{\circ}{E}_{(a)}^A = \sum_b R_{(a)(b)}(\alpha_{(e)}) {}^4\overset{\circ}{E}_{(b)}^A$, ${}^3e_{(a)}^r = \sum_b R_{(a)(b)}(\alpha_{(e)}) {}^3\bar{e}_{(b)}^r$, where $\alpha_{(a)}(\tau, \vec{\sigma})$ are three point-dependent Euler angles. After having chosen an arbitrary point-dependent origin $\alpha_{(a)}(\tau, \vec{\sigma}) = 0$, we arrive at the following adapted tetrads and cotetrads [$\bar{n}_{(a)} = \sum_b n_{(b)} R_{(b)(a)}(\alpha_{(e)})$, $\sum_a n_{(a)} {}^3e_{(a)}^r = \sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)}^r$]

$${}^4\overset{\circ}{E}_{(o)}^A = {}^4\overset{\circ}{E}_{(o)}^A = \frac{1}{1+n} (1; -\sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)}^r) = l^A, \quad {}^4\overset{\circ}{E}_{(a)}^A = (0; {}^3\bar{e}_{(a)}^r),$$

⁹ In each tangent plane to a point of Σ_τ the point-dependent standard Wigner boost for time-like Poincare' orbits $L^{(\alpha)}_{(\beta)}(V(z(\sigma))); \overset{\circ}{V} = \delta^{(\alpha)}_{(\beta)} + 2\epsilon V^{(\alpha)}(z(\sigma)) \overset{\circ}{V}_{(\beta)} - \epsilon \frac{(V^{(\alpha)}(z(\sigma)) + \overset{\circ}{V}^{(\alpha)}) (V_{(\beta)}(z(\sigma)) + \overset{\circ}{V}_{(\beta)})}{1 + V^{(\alpha)}(z(\sigma))} \stackrel{def}{=} L^{(\alpha)}_{(\beta)}(\varphi_{(a)})$

sends the unit future-pointing time-like vector $\overset{\circ}{V}^{(\alpha)} = (1; 0)$ into the unit time-like vector $V^{(\alpha)} = {}^4E_A^{(\alpha)} l^A = \left(\sqrt{1 + \sum_a \varphi_{(a)}^2}; \varphi^{(a)} = -\epsilon \varphi_{(a)} \right)$, where l^A is the unit future-pointing normal to Σ_τ . We have $L^{-1}(\varphi_{(a)}) = {}^4\eta L^T(\varphi_{(a)}) {}^4\eta = L(-\varphi_{(a)})$. As a consequence, the flat indices (a) of the adapted tetrads and cotetrads and of the triads and cotriads on Σ_τ transform as Wigner spin-1 indices under point-dependent $\text{SO}(3)$ Wigner rotations $R_{(a)(b)}(V(z(\sigma)); \Lambda(z(\sigma)))$ associated with Lorentz transformations $\Lambda^{(\alpha)}_{(\beta)}(z)$ in the tangent plane to the space-time in the given point of Σ_τ . Instead the index (o) of the adapted tetrads and cotetrads is a local Lorentz scalar index.

¹⁰ Since we use the positive-definite 3-metric $\delta_{(a)(b)}$, we shall use only lower flat spatial indices. Therefore for the cotriads we use the notation ${}^3e_r^{(a)} \stackrel{def}{=} {}^3e_{(a)r}$ with $\delta_{(a)(b)} = {}^3e_{(a)}^r {}^3e_{(b)r}$.

$${}^4\overset{\circ}{E}_A^{(o)} = {}^4\overset{\circ}{E}_A^{(o)} = (1+n)(1; \vec{0}) = \epsilon l_A, \quad {}^4\overset{\circ}{E}_A^{(a)} = (\bar{n}_{(a)}; {}^3\bar{e}_{(a)r}), \quad (3.2)$$

which we shall use as a reference standard.

The expressions for the general tetrad and for the 4-metric ¹¹ are

$$\begin{aligned} {}^4E_{(\alpha)}^A &= {}^4\overset{\circ}{E}_{(\beta)}^A L^{(\beta)}_{(\alpha)}(\varphi_{(a)}) = {}^4\overset{\circ}{E}_{(o)}^A L^{(o)}_{(\alpha)}(\varphi_{(c)}) + \sum_{ab} {}^4\overset{\circ}{E}_{(b)}^A R_{(b)(a)}^T(\alpha_{(c)}) L^{(a)}_{(\alpha)}(\varphi_{(c)}), \\ {}^4g_{AB} &= {}^4\overset{\circ}{E}_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} {}^4\overset{\circ}{E}_B^{(\beta)} = {}^4\overset{\circ}{E}_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} {}^4\overset{\circ}{E}_B^{(\beta)}, \\ {}^4g_{\tau\tau} &= \epsilon[(1+n)^2 - {}^3g^{rs}n_r n_s] = \epsilon[(1+n)^2 - \sum_a \bar{n}_{(a)}^2], \quad {}^4g_{\tau r} = -\epsilon n_r = -\epsilon \sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\ {}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \sum_a {}^3e_{(a)r} {}^3e_{(a)s} = -\epsilon \sum_a {}^3\bar{e}_{(a)r} {}^3\bar{e}_{(a)s}, \\ \sqrt{-g} &= \sqrt{|{}^4g|} = \frac{\sqrt{{}^3g}}{\sqrt{\epsilon {}^4g^{\tau\tau}}} = \sqrt{\gamma}(1+n) = {}^3e(1+n), \quad {}^3g = \gamma = ({}^3e)^2, \quad {}^3e = \det {}^3e_{(a)r}. \end{aligned} \quad (3.3)$$

The future-oriented unit normal to Σ_τ and the projector on Σ_τ are $l_A = \epsilon(1+n)(1; 0)$, ${}^4g^{AB}l_A l_B = \epsilon$, $l^A = \epsilon(1+n){}^4g^{A\tau} = \frac{1}{1+n}(1; -n^r) = \frac{1}{1+n}(1; -\sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)}^r)$, ${}^3h_A^B = \delta_A^B - \epsilon l_A l^B$.

Each 3+1 splitting of (either Minkowski or asymptotically Minkowskian) space-time, i.e. each global non-inertial frame, has two associated congruences of time-like observers:

i) The congruence of the Eulerian observers with the unit normal $l^\mu(\tau, \vec{\sigma}) = (z_A^\mu l^A)(\tau, \vec{\sigma})$ to the 3-spaces as unit 4-velocity. The world-lines of these observers are the integral curves of the unit normal and in general are not geodesics. In adapted radar 4-coordinates the Eulerian observers carry the contro-variant $(l^A(\tau, \vec{\sigma}), {}^4\overset{\circ}{E}_{(a)}^A(\tau, \vec{\sigma}))$ and covariant $(l_A(\tau, \vec{\sigma}), {}^4\overset{\circ}{E}_{(a)A}(\tau, \vec{\sigma}))$ orthonormal tetrads defined in of Eqs.(3.2).

ii) The skew congruence with unit 4-velocity $v^\mu(\tau, \vec{\sigma}) = (z_A^\mu v^A)(\tau, \vec{\sigma})$ (in general it is not surface-forming, i.e. it has a non-vanishing vorticity). The observers of the skew congruence have the world-lines (integral curves of the 4-velocity) defined by $\sigma^r = \text{const.}$ for every τ , because the unit 4-velocity tangent to the flux lines $x_{\vec{\sigma}_o}^\mu(\tau) = z^\mu(\tau, \vec{\sigma}_o)$ is $v_{\vec{\sigma}_o}^\mu(\tau) = z_\tau^\mu(\tau, \vec{\sigma}_o)/\sqrt{\epsilon {}^4g_{\tau\tau}(\tau, \vec{\sigma}_o)}$. They carry the adapted contro-variant and covariant orthonormal tetrads $(\mathcal{V}_{(a)}^A(\tau, \vec{\sigma}))$ are not tangent to the 3-spaces Σ_τ like ${}^4\overset{\circ}{E}_{(a)}^A(\tau, \vec{\sigma})$ of Eqs.(3.2)

¹¹ The 3-metric ${}^3g_{rs}$ has signature $(+++)$, so that we may put all the flat 3-indices *down*. We have ${}^3g^{ru} {}^3g_{us} = \delta_s^r$.

$$\begin{aligned}
v^A(\tau, \vec{\sigma}) &= \frac{(1; 0)}{\sqrt{(1+n)^2 - \sum_a \bar{n}_{(a)}^2}}(\tau, \vec{\sigma}), \\
\mathcal{V}_{(a)}^A(\tau, \vec{\sigma}) &= \left(\frac{\bar{n}_{(a)}}{(1+n)^2}; \sum_b \left(\delta_{(a)(b)} - \frac{\bar{n}_{(a)} \bar{n}_{(b)}}{(1+n)^2} \right) {}^3\bar{e}_{(b)}^r \right)(\tau, \vec{\sigma}), \\
{}^\epsilon v_A(\tau, \vec{\sigma}) &= \left(\sqrt{(1+n)^2 - \sum_c \bar{n}_{(c)}^2}; \frac{-\bar{n}_{(a)} {}^3\bar{e}_{(a)r}}{\sqrt{(1+n)^2 - \sum_c \bar{n}_{(c)}^2}} \right)(\tau, \vec{\sigma}), \\
\mathcal{V}_{(a)A}(\tau, \vec{\sigma}) &= (0; {}^3\bar{e}_{(a)r})(\tau, \vec{\sigma}).
\end{aligned} \tag{3.4}$$

The 16 configurational variables in the ADM action are $\varphi_{(a)}$, $1+n$, $n_{(a)}$, ${}^3e_{(a)r}$. There are ten primary constraints (the vanishing of the 7 momenta of boosts, lapse and shift variables plus three constraints describing the rotation on the flat indices (a) of the cotriads) and four secondary ones (the super-Hamiltonian and super-momentum constraints): all of them are first class in the phase space spanned by 16+16 fields. This implies that there are 14 gauge variables describing *inertial effects* and 2 canonical pairs of physical degrees of freedom describing the *tidal effects* of the gravitational field (namely gravitational waves in the weak field limit). In this canonical basis only the momenta ${}^3\pi_{(a)}^r$ conjugated to the cotriads are not vanishing. The basis of canonical variables for this formulation of tetrad gravity, naturally adapted to 7 of the 14 first-class constraints, is

$\varphi_{(a)}$	n	$\bar{n}_{(a)}$	${}^3e_{(a)r}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi_n \approx 0$	$\pi_{n_{(a)}} \approx 0$	${}^3\pi_{(a)}^r$

(3.5)

In the next Section I will show a more convenient canonical basis in which the configuration variables are $\alpha_{(a)}$, $\varphi_{(a)}$, $1+n$, $\bar{n}_{(a)}$, ${}^3\bar{e}_{(a)r}$.

From Eqs.(5.5) of Ref.[7] we assume the following (direction-independent, so to kill super-translations) boundary conditions at spatial infinity ($r = \sqrt{\sum_r (\sigma^r)^2}$; $\epsilon > 0$; $M = \text{const.}$): $n(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-(2+\epsilon)})$, $\pi_n(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-3})$, $n_{(a)}(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-\epsilon})$, $\pi_{n_{(a)}}(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-3})$, $\varphi_{(a)}(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-(1+\epsilon)})$, $\pi_{\varphi_{(a)}}(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-2})$, ${}^3e_{(a)r}(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} \left(1 + \frac{M}{2r}\right) \delta_{ar} + O(r^{-3/2})$, ${}^3\pi_{(a)}^r(\tau, \sigma^r) \rightarrow_{r \rightarrow \infty} O(r^{-5/2})$.

IV. THE YORK CANONICAL BASIS

In Ref.[?] a canonical transformation to a canonical basis adapted to ten of the first class constraints was found. It implements the York map of Ref.[31] (in the cases in which the 3-metric ${}^3g_{rs}$ has three distinct eigenvalues) and diagonalizes the York-Lichnerowicz approach. Its final form is $(\alpha_{(a)}(\tau, \sigma^r))$ are the Euler angles of the previous Section; $V_{ua} \stackrel{def}{=} \sum_v V_{uv} \delta_{v(a)}$

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$	θ^r	$\tilde{\phi}$	$R_{\bar{a}}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi_{\alpha_{(a)}} \approx 0$	$\pi_n \approx 0$	$\pi_{\bar{n}_{(a)}} \approx 0$	$\pi_r^{(\theta)}$	$\pi_{\tilde{\phi}} = \frac{c^3}{12\pi G} {}^3K$	$\Pi_{\bar{a}}$

$$\begin{aligned}
{}^3e_{(a)r} &= \sum_b R_{(a)(b)}(\alpha_{(c)}) {}^3\bar{e}_{(b)r}, & {}^3\bar{e}_{(a)r} &= V_{ra}(\theta^i) \tilde{\phi}^{1/3} e^{\sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}}}, \\
{}^4g_{\tau\tau} &= \epsilon [(1+n)^2 - \sum_a \bar{n}_{(a)}^2], & {}^4g_{\tau r} &= -\epsilon \sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\
{}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \tilde{\phi}^{2/3} \sum_a V_{ra}(\theta^i) V_{sa}(\theta^i) Q_a^2, & Q_a &= e^{\sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}}},
\end{aligned} \tag{4.1}$$

The set of numerical parameters $\gamma_{\bar{a}a}$ satisfies [5] $\sum_u \gamma_{\bar{a}u} = 0$, $\sum_u \gamma_{\bar{a}u} \gamma_{\bar{b}u} = \delta_{\bar{a}\bar{b}}$, $\sum_{\bar{a}} \gamma_{\bar{a}u} \gamma_{\bar{a}v} = \delta_{uv} - \frac{1}{3}$. Each solution of these equations defines a different York canonical basis.

This canonical basis has been found due to the fact that the 3-metric ${}^3g_{rs}$ is a real symmetric 3×3 matrix, which may be diagonalized with an *orthogonal* matrix $V(\theta^r)$, $V^{-1} = V^T$ ($\sum_u V_{ua} V_{ub} = \delta_{ab}$, $\sum_a V_{ua} V_{va} = \delta_{uv}$, $\sum_{uv} \epsilon_{uvw} V_{ua} V_{vb} = \sum_c \epsilon_{abc} V_{cw}$), $\det V = 1$, depending on three parameters θ^r . If we choose these three gauge parameters to be Euler angles $\hat{\theta}^i(\tau, \vec{\sigma})$, we get a description of the 3-coordinate systems on Σ_τ from a local point of view, because they give the orientation of the tangents to the three 3-coordinate lines through each point. However, for the calculations (see Refs.[24]) it is more convenient to choose the three gauge parameters as first kind coordinates $\theta^i(\tau, \vec{\sigma})$ ($-\infty < \theta^i < +\infty$) on the $O(3)$ group manifold, so that by definition we have $V_{ru}(\theta^i) = \left(e^{-\sum_i \hat{T}_i \theta^i} \right)_{ru}$, where $(\hat{T}_i)_{ru} = \epsilon_{ru i}$ are the generators of the $\mathfrak{o}(3)$ Lie algebra in the adjoint representation, and the Euler angles may be expressed as $\hat{\theta}^i = f^i(\theta^n)$. The Cartan matrix has the form $A(\theta^n) = \frac{1 - e^{-\sum_i \hat{T}_i \theta^i}}{\sum_i \hat{T}_i \theta^i}$ and satisfies $A_{ri}(\theta^n) \theta^i = \delta_{ri}$; $B(\theta^i) = A^{-1}(\theta^i)$.

From now on for the sake of notational simplicity we shall use $\vec{\sigma}$ for the curvilinear coordinates σ^r and V for $V(\theta^i)$.

This canonical transformation realizes a *York map* because the gauge variable $\pi_{\tilde{\phi}}$ (describing the freedom in the choice of the trace of the extrinsic curvature of the instantaneous 3-spaces Σ_τ) is proportional to *York internal extrinsic time* 3K . It is the only gauge variable among the momenta: this is a reflex of the Lorentz signature of space-time, because $\pi_{\tilde{\phi}}$ and θ^n can be used as a set of 4-coordinates [46]. The York time describes the effect of gauge transformations producing a deformation of the shape of the 3-space along the 4-normal to the 3-space as a 3-sub-manifold of space-time.

Its conjugate variable, to be determined by the super-Hamiltonian constraint, is $\tilde{\phi} = {}^3\bar{e} = \sqrt{\det {}^3g_{rs}}$, which is proportional to *Misner's internal intrinsic time*; moreover $\tilde{\phi}$ is the *3-volume density* on Σ_τ : $V_R = \int_R d^3\sigma \tilde{\phi}$, $R \subset \Sigma_\tau$. Since we have ${}^3g_{rs} = \tilde{\phi}^{2/3} {}^3\hat{g}_{rs}$ with $\det {}^3\hat{g}_{rs} = 1$, $\tilde{\phi}$ is also called the *conformal factor* of the 3-metric.

The two pairs of canonical variables $R_{\bar{a}}, \Pi_{\bar{a}}$, $\bar{a} = 1, 2$, describe the generalized *tidal effects*, namely the independent physical degrees of freedom of the gravitational field. They are 3-scalars on Σ_τ and the configuration tidal variables $R_{\bar{a}}$ depend *only on the eigenvalues of the 3-metric*. They are DO's *only* with respect to the gauge transformations generated by 10 of the 14 first class constraints. Let us remark that, if we fix completely the gauge and we go to Dirac brackets, then the only surviving dynamical variables $R_{\bar{a}}$ and $\Pi_{\bar{a}}$ become two pairs of *non canonical* Dirac observables for that gauge: the two pairs of canonical Dirac observables have to be found as a Darboux basis of the copy of the reduced phase space identified by the gauge and they will be (in general non-local) functionals of the $R_{\bar{a}}, \Pi_{\bar{a}}$ variables.

Since the variables $\tilde{\phi}$ and $\pi_i^{(\theta)}$ are determined by the super-Hamiltonian (i.e. the Lichnerowicz equation) and super-momentum constraints respectively, the *arbitrary gauge variables* are $\alpha_{(a)}, \varphi_{(a)}, \theta^i, \pi_{\tilde{\phi}}, n$ and $\bar{n}_{(a)}$. As shown in Refs.[?], they describe the following generalized *inertial effects*:

a) $\alpha_{(a)}(\tau, \vec{\sigma})$ and $\varphi_{(a)}(\tau, \vec{\sigma})$ are the 6 configuration variables parametrizing the $O(3,1)$ gauge freedom in the choice of the tetrads in the tangent plane to each point of Σ_τ and describe the arbitrariness in the choice of a tetrad to be associated to a time-like observer, whose world-line goes through the point $(\tau, \vec{\sigma})$. They fix *the unit 4-velocity of the observer and the conventions for the orientation of three gyroscopes and their transport along the world-line of the observer*. The *Schwinger time gauges* are defined by the gauge fixings $\alpha_{(a)}(\tau, \vec{\sigma}) \approx 0$, $\varphi_{(a)}(\tau, \vec{\sigma}) \approx 0$.

b) $\theta^i(\tau, \vec{\sigma})$ (depending only on the 3-metric) describe the arbitrariness in the choice of the 3-coordinates in the instantaneous 3-spaces Σ_τ of the chosen non-inertial frame centered on an arbitrary time-like observer. Their choice will induce a pattern of *relativistic inertial forces* for the gravitational field, whose potentials are the functions $V_{ra}(\theta^i)$ present in the weak ADM energy \hat{E}_{ADM} .

c) $\bar{n}_{(a)}(\tau, \vec{\sigma})$, the shift functions, describe which points on different instantaneous 3-spaces have the same numerical value of the 3-coordinates. They are the inertial potentials describing the effects of the non-vanishing off-diagonal components ${}^4g_{\tau r}(\tau, \vec{\sigma})$ of the 4-metric, namely they are the *gravito-magnetic potentials*¹² responsible of effects like the dragging of inertial frames (Lens-Thirring effect) in the post-Newtonian approximation. The shift functions are determined by the τ -preservation of the gauge fixings determining the gauge variables $\theta^i(\tau, \vec{\sigma})$.

d) $\pi_{\tilde{\phi}}(\tau, \vec{\sigma})$, i.e. the York time ${}^3K(\tau, \vec{\sigma})$, describes the non-dynamical arbitrariness in the choice of the convention for the synchronization of distant clocks which remains in the

¹² In the post-Newtonian approximation in harmonic gauges they are the counterpart of the electro-magnetic vector potentials describing magnetic fields [32]: A) $N = 1 + n$, $n \stackrel{def}{=} -\frac{4\epsilon}{c^2} \Phi_G$ with Φ_G the *gravito-electric potential*; B) $n_r \stackrel{def}{=} \frac{2\epsilon}{c^2} A_{Gr}$ with A_{Gr} the *gravito-magnetic potential*; C) $E_{Gr} = \partial_r \Phi_G - \partial_\tau (\frac{1}{2} A_{Gr})$ (the *gravito-electric field*) and $B_{Gr} = \epsilon_{ruv} \partial_u A_{Gv} = c \Omega_{Gr}$ (the *gravito-magnetic field*). Let us remark that in arbitrary gauges the analogy with electro-magnetism breaks down.

transition from SR to GR. Since the York time is present in the Dirac Hamiltonian, it is a *new inertial potential* connected to the problem of the relativistic freedom in the choice of the *shape of the instantaneous 3-space*, which has no Newtonian analogue (in Galilei space-time time is absolute and there is an absolute notion of Euclidean 3-space). Its effects are completely unexplored.

e) $1 + n(\tau, \vec{\sigma})$, the lapse function appearing in the Dirac Hamiltonian, describes the arbitrariness in the choice of the unit of proper time in each point of the simultaneity surfaces Σ_τ , namely how these surfaces are packed in the 3+1 splitting. The lapse function is determined by the τ -preservation of the gauge fixing for the gauge variable ${}^3K(\tau, \vec{\sigma})$.

The extrinsic curvature tensor of the 3-space Σ_τ has the expression

$$\begin{aligned} {}^3K_{rs} = & -\frac{4\pi G}{c^3} \tilde{\phi}^{-1/3} \left(\sum_a Q_a^2 V_{ra} V_{sa} [2 \sum_{\bar{b}} \gamma_{\bar{b}a} \Pi_{\bar{b}} - \tilde{\phi} \pi_{\tilde{\phi}}] + \right. \\ & \left. + \sum_{ab} Q_a Q_b (V_{ra} V_{sb} + V_{rb} V_{sa}) \sum_{twi} \frac{\epsilon_{abt} V_{wt} B_{iw} \pi_i^{(\theta)}}{Q_b Q_a^{-1} - Q_a Q_b^{-1}} \right). \end{aligned} \quad (4.2)$$

As shown in Eqs.(2.11)-(2.16) of the first paper in Refs.[24], if we use radar 4-coordinates, the covariant unit normal $\epsilon l_A = (1 + n)(1; 0)$, i.e. the 4-velocity of the Eulerian observers, has the following covariant derivative

$${}^4\nabla_A \epsilon l_B = \epsilon l_A {}^3a_B + \sigma_{AB} + \frac{1}{3} \theta ({}^4g_{AB} - \epsilon l_A l_B) - \omega_{AB}. \quad (4.3)$$

The quantities appearing in Eqs.(4.3) are:

- 1) the *acceleration* 3a_A of the Eulerian observers (${}^3a_r = \partial_r \ln(1 + n)$, ${}^3a_\tau = {}^3a_r {}^3\bar{e}_{(a)}^r \bar{n}_{(a)}$);
- 2) their *expansion*, which coincides with the *York external time*¹³ : $\theta = {}^4\nabla_A l^A = -\epsilon {}^3K = -\epsilon \frac{12\pi G}{c^3} \pi_{\tilde{\phi}}$;
- 3) their *shear* σ_{AB} , whose components $\sigma_{(\alpha)(\beta)}$ along the tetrads (5.2) turn out to be $\sigma_{(o)(o)} = \sigma_{(o)(a)} = 0$ and $\sigma_{(a)(b)} = \sigma_{(b)(a)} = ({}^3K_{rs} - \frac{1}{3} {}^3g_{rs} {}^3K) {}^3\bar{e}_{(a)}^r {}^3\bar{e}_{(b)}^s$ with $\sum_a \sigma_{(a)(a)} = 0$. $\sigma_{(a)(b)}$ depends upon θ^r , $\tilde{\phi}$, $R_{\bar{a}}$, $\pi_r^{(\theta)}$ and $\Pi_{\bar{a}}$.

Instead the definition of Eulerian observers implies that their *vorticity* or *twist* vanishes because the congruence is surface-forming: $\omega_{AB} = -\omega_{BA} = 0$.

Then the following results can be obtained

$$\tilde{\phi} \sigma_{(a)(a)} = -\frac{8\pi G}{c^3} \sum_{\bar{a}} \gamma_{\bar{a}a} \Pi_{\bar{a}}, \rightarrow \Pi_{\bar{a}} = -\frac{c^3}{8\pi G} \tilde{\phi} \sum_a \gamma_{\bar{a}a} \sigma_{(a)(a)},$$

¹³ In cosmology it is proportional to the *Hubble parameter* $H = \frac{1}{3} \theta$ and determines the dimensionless (cosmological) *deceleration parameter* $q = -3 \theta^{-2} l^A \partial_A \theta - 1$

$$\begin{aligned}
\tilde{\phi} \sigma_{(a)(b)}|_{a \neq b} &= -\frac{8\pi G}{c^3} \sum_{tw} \frac{\epsilon_{abt} V_{wt}}{Q_b Q_a^{-1} - Q_a Q_b^{-1}} \sum_i B_{iw} \pi_i^{(\theta)}, \\
\Rightarrow \quad \pi_i^{(\theta)} &= \frac{c^3}{8\pi G} \tilde{\phi} \sum_{wtab} A_{wi} V_{wt} Q_a Q_b^{-1} \epsilon_{tab} \sigma_{(a)(b)}|_{a \neq b}, \\
{}^3K_{rs} &= -\frac{\epsilon}{3} {}^3g_{rs} \theta + \sigma_{(a)(b)} {}^3\bar{e}_{(a)r} {}^3\bar{e}_{(b)s}.
\end{aligned} \tag{4.4}$$

Therefore the diagonal elements of the shear of the Eulerian observers describe the tidal momenta $\Pi_{\bar{a}}$, while the non-diagonal elements determine the variables $\pi_i^{(\theta)}$, determined by the super-momentum constraints. Moreover their expansion θ is the inertial gauge variable determining the non-dynamical part (general relativistic gauge freedom in clock synchronization) of the shape of the instantaneous 3-spaces Σ_τ .

See the first paper in Refs.[24] for the expression of the super-momentum constraints $\mathcal{H}_{(a)}(\tau, \vec{\sigma}) \approx 0$ [Eqs.(3.41)-(3.42)] and of the super-Hamiltonian constraint $\mathcal{H}(\tau, \vec{\sigma}) \approx 0$ (the Lichnerowicz equation) [Eqs.(3.44)-(3.45)]. The weak ADM energy is given in Eqs. (3.43)-(3.45) of that paper (while the other weak Poincaré generators are given in Eqs.(3.47)): in it there is a negative kinetic term proportional to $({}^3K)^2$ (the York time is a momentum!), vanishing only in the gauges ${}^3K(\tau, \vec{\sigma}) = 0$. It comes from the bilinear in momenta present both in the super-Hamiltonian and in the weak ADM energy: it was known that this quadratic form was not definite positive but only in the York canonical basis this can be made explicit. The expression of the weak ADM energy in terms of the expansion ($\theta = -\epsilon {}^3K = -\epsilon \frac{12\pi G}{c^3} \pi_{\tilde{\phi}}$) and shear of the Eulerian observers is

$$\begin{aligned}
\hat{E}_{ADM} &= c \int d^3\sigma \left[\tilde{\mathcal{M}} - \frac{c^3}{16\pi G} \mathcal{S} + \frac{4\pi G}{c^3} \tilde{\phi}^{-1} \sum_{\bar{b}} \Pi_{\bar{b}}^2 + \right. \\
&\quad \left. + \tilde{\phi} \left(\frac{c^3}{16\pi G} \sum_{ab, a \neq b} \sigma_{(a)(b)}^2 - \frac{6\pi G}{c^3} \pi_{\tilde{\phi}}^2 \right) \right] (\tau, \vec{\sigma}),
\end{aligned} \tag{4.5}$$

where $\tilde{\mathcal{M}} = \tilde{\phi} (1+n)^2 T^{\tau\tau}$ is the energy-mass density of the matter (with energy-momentum tensor T^{AB}) and $\mathcal{S}(\tilde{\phi}, \theta^i, R_{\bar{a}})$ is an inertial potential depending on the choice of the 3-coordinates in the 3-space (it is the $\Gamma - \Gamma$ term in the scalar 3-curvature of the 3-space).

Finally the Dirac Hamiltonian is

$$\begin{aligned}
H_D &= \frac{1}{c} \hat{E}_{ADM} + \int d^3\sigma \left[n \mathcal{H} - \bar{n}_{(a)} \mathcal{H}_{(a)} \right] (\tau, \vec{\sigma}) + \lambda_r(\tau) \hat{P}_{ADM}^r + \\
&\quad + \int d^3\sigma \left[\lambda_n \pi_n + \lambda_{\bar{n}_{(a)}} \pi_{\bar{n}_{(a)}} + \lambda_{\varphi_{(a)}} \pi_{\varphi_{(a)}} + \lambda_{\alpha_{(a)}} \pi_{\alpha_{(a)}}^{(\alpha)} \right] (\tau, \vec{\sigma}),
\end{aligned} \tag{4.6}$$

where the $\lambda_{...}(\tau, \vec{\sigma})$'s are Dirac multipliers. In particular the Dirac multiplier $\lambda_r(\tau)$ implements the rest frame condition $\hat{P}_{ADM}^r \approx 0$.

In the York canonical basis, where the super-momentum and super-Hamiltonian constraints are coupled *elliptic* equations on the 3-space Σ_τ , the Hamilton equations generated by this Dirac Hamiltonian (replacing the standard 12 ADM equations and the matter equations ${}^4\nabla_A T^{AB} = 0$) are divided in four groups:

A) the contracted Bianchi identities, namely the evolution equations for the solutions $\tilde{\phi}$ and $\pi_i^{(\theta)}$ of the constraints (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times);

B) the evolution equation for the four basic gauge variables θ^i and 3K : these equations determine the lapse and the shift functions once four gauge-fixings for the basic gauge variables are given;

C) the *hyperbolic* evolution equations for the tidal variables $R_{\bar{a}}$, $\Pi_{\bar{a}}$;

D) the Hamilton equations for matter, when present.

Once a gauge is completely fixed by giving the six gauge-fixings for the $O(3,1)$ variables $\varphi_{(a)}$, $\alpha_{(a)}$ (choice of the tetrads and of their transport) and four gauge-fixings for θ^i (choice of the 3-coordinates on the 3-space) and 3K (determination of the shape of the 3-space as a 3-sub-manifold of space-time by means of a clock synchronization convention), the Hamilton equations become a deterministic set of coupled PDE's for the lapse and shift functions (secondary inertial gauge variables ¹⁴), the tidal variables and the matter. Given a solution of the super-momentum and super-Hamiltonian constraints and the Cauchy data for the tidal variables on an initial 3-space, we can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer in the chosen gauge.

In Refs.[39] there is the Hamiltonian expression of radar tensors which coincide with the Riemann and Weyl tensors, expressed as radar tensors, on the solutions of Einstein equations. Moreover, by using the time-like normal to the 3-spaces and the space-like direction identified by the shift function (for solutions of Einstein equations in which it is not identically zero) it is possible to build null tetrads and to give the Hamiltonian formulation of the Newman-Penrose formalism [40]. Therefore we get the Hamiltonian expression of Ricci and Weyl scalars and of the eigenvalues of the Weyl tensor. It is shown that the Bergmann observables [47, 48], [46], built with these eigenvalues cannot be DO's.

V. THE NON-HARMONIC 3-ORTHOGONAL SCHWINGER TIME GAUGES, THE POST-MINKOWSKIAN LINEARIZATION AND GRAVITATIONAL WAVES

In Refs.[24] the family of *3-orthogonal Schwinger time gauges* defined by the following gauge fixings ($F(\tau, \sigma^r)$ is an arbitrary numerical function)

$$\begin{aligned}\varphi_{(a)}(\tau, \sigma^r) &\approx 0, & \alpha_{(a)}(\tau, \sigma^r) &\approx 0, \\ \theta^i(\tau, \sigma^r) &\approx 0, & {}^3K(\tau, \sigma^r) &\approx F(\tau, \sigma^r),\end{aligned}\tag{5.1}$$

is defined and studied. In these gauges the instantaneous Riemannian 3-spaces Σ_τ have a non-fixed trace 3K of the extrinsic curvature but a diagonal 3-metric ${}^4g_{rs} = -\epsilon {}^3g_{rs} \approx -\epsilon \tilde{\phi}^{2/3} Q_r^2 \delta_{rs}$ (with $Q_r = e \sum_{\bar{a}} \gamma_{\bar{a}a} R_{\bar{a}}$, see Eqs.(4.1)).

¹⁴ As said in B) their gauge fixings are induced by those for primary basic gauge variables. This is a consequence of Dirac theory of constraints [27]. Instead in numerical gravity one gives independent gauge fixings for both the primary and secondary gauge variables in such a way to minimize the computer time.

These gauges are *not harmonic gauges*. Their main property is that in them the equations for the lapse and shift variables (see B) of the previous Section) are *elliptic* PDE's inside the 3-space like the constraints. Instead, as shown in Section V of the first paper in Refs.[24], the analogous equations in the family of harmonic gauges are *hyperbolic* PDE's like for the tidal variables. Therefore in harmonic gauges both the tidal variables and the lapse and shift functions depend (in a retarded way) from the *no-incoming radiation* condition on the Cauchy surface in the past (so that the knowledge of 3K from the initial time till today is needed).

Instead in the family of 3-orthogonal gauges only the tidal variables (the gravitational waves after linearization), and therefore the 3-metric inside Σ_τ , depend (in a retarded way) on the no-incoming radiation condition. The solutions $\tilde{\phi}$ and $\pi_i^{(\theta)}$ (or $\sigma_{(a)(b)}|_{a \neq b}$) of the constraints and the lapse $1+n$ and shift $\bar{n}_{(a)}$ functions depend only on the 3-space Σ_τ with fixed τ . If the matter consists of positive energy particles (with a Grassmann regularization of the gravitational self-energies) [24] these solutions will contain action-at-a-distance gravitational potentials (replacing the Newton ones) and gravito-magnetic potentials.

In the first paper of Ref.[24], we studied the coupling of N charged scalar particles plus the electro-magnetic field to ADM tetrad gravity in the class of asymptotically Minkowskian space-times without super-translations. To regularize the electro-magnetic and gravitational self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued ¹⁵.

The introduction of the non-covariant radiation gauge (see Ref.[21] for the special relativistic version) allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the action-at-a-distance Coulomb interaction among the particles in the non-flat Riemannian instantaneous 3-spaces of global non-inertial frames.

After the reformulation of the whole system in the York canonical basis, we give the restriction of the Hamilton equations and of the constraints to the family of *non-harmonic 3-orthogonal* Schwinger time gauges.

Then in the second paper of Ref.[24] it was shown that in this family of non-harmonic 3-orthogonal Schwinger gauges it is possible to define a consistent *linearization* of ADM canonical tetrad gravity plus matter in the weak field approximation, to obtain a formulation of *Hamiltonian Post-Minkowskian gravity with non-flat Riemannian 3-spaces and asymptotic Minkowski background*.

This means that the 4-metric tends to the asymptotic Minkowski metric at spatial infinity, ${}^4g_{AB} \rightarrow {}^4\eta_{AB}$. The decomposition ${}^4g_{AB} = {}^4\eta_{AB} + {}^4h_{(1)AB}$, with a first order perturbation ${}^4h_{(1)AB}$ vanishing at spatial infinity, is only used for calculations, but has no intrinsic meaning because the 3-spaces Σ_τ have a first order derivation from Euclidean 3-spaces. Instead in the

¹⁵ Both quantities are two-valued. The elementary electric charges are $Q = \pm e$, with e the electron charge. Analogously the sign of the energy of a particle is a topological two-valued number (the two branches of the mass-shell hyperboloid). The formal quantization of these Grassmann variables gives two-level fermionic oscillators. At the classical level the self-energies make the classical equations of motion ill-defined on the world-lines of the particles. The ultraviolet and infrared Grassmann regularization allows to cure this problem and to get consistent solution of regularized equations of motion. See Refs.[49] for the electro-magnetic case.

standard linearization one introduces a fixed Minkowski background space-time, introduces the decomposition ${}^4g_{\mu\nu}(x) = {}^4\eta_{\mu\nu} + {}^4h_{\mu\nu}(x)$ and studies the linearized equations of motion for the small Minkowskian fields ${}^4h_{\mu\nu}(x)$. The approximation is assumed valid over a *big enough characteristic length L interpretable as the reduced wavelength $\lambda/2\pi$ of the resulting GW's* (only for distances higher of L the linearization breaks down and curved space-time effects become relevant). See Ref.[50] for a review of all the results of the standard approach and of the existing points of view (Damour, Will,...) on the subject (see also Appendix A of the second paper in Refs.[24]).

If $\zeta \ll 1$ is a small a-dimensional parameter, the consistent Hamiltonian linearization implies the following restrictions on the variables of the York canonical basis in the family of 3-orthogonal gauges with ${}^3K = F$ (the tidal variables $R_{\bar{a}}$ are slowly varying over the length L and times L/c ; we have $(\frac{L}{4\mathcal{R}})^2 = O(\zeta)$, where ${}^4\mathcal{R}$ is the mean radius of curvature of space-time)

$$\begin{aligned}
R_{\bar{a}}(\tau, \vec{\sigma}) &= R_{(1)\bar{a}}(\tau, \vec{\sigma}) = O(\zeta) \ll 1, & \Pi_{\bar{a}}(\tau, \vec{\sigma}) &= \Pi_{(1)\bar{a}}(\tau, \vec{\sigma}) = \frac{1}{LG} O(\zeta), \\
\tilde{\phi} &= \sqrt{\det {}^3g_{rs}} = 1 + 6\phi_{(1)} + O(\zeta^2), & \bar{n}_{(r)} &= -\epsilon {}^4g_{rr} = \bar{n}_{(1)(r)} + O(\zeta^2), \\
N &= 1 + n = 1 + n_{(1)} + O(\zeta^2), & \epsilon {}^4g_{\tau\tau} &= 1 + 2n_{(1)} + O(\zeta^2), \\
{}^3K &= \frac{12\pi G}{c^3} \pi_{\tilde{\phi}} = {}^3K_{(1)} = \frac{12\pi G}{c^3} \pi_{(1)\tilde{\phi}} = \frac{1}{L} O(\zeta), & \sigma_{(a)(b)}|_{a \neq b} &= \sigma_{(1)(a)(b)}|_{a \neq b} = \frac{1}{L} O(\zeta), \\
{}^3g_{rs} &= -\epsilon {}^4g_{rs} = [1 + 2(\Gamma_r^{(1)} + 2\phi_{(1)})] \delta_{rs} + O(\zeta^2), & \Gamma_a^{(1)} &= \sum_{\bar{a}r} \gamma_{\bar{a}a} R_{\bar{a}}.
\end{aligned} \tag{5.2}$$

The particles (whose coinciding inertial and gravitational mass is m_i) are described by 3-coordinates $\eta_i^r(\tau)$ (the radar 3-coordinates of the intersection of the world-line with the 3-space: $x^\mu(\tau) = z^\mu(\tau, \eta_i^r(\tau))$) and by 3-momenta $\kappa_{ir}(\tau)$. See Refs.[24] for the description of the electro-magnetic field. The consistency of the Hamiltonian linearization requires the introduction of a *ultra-violet cutoff M for matter* such that $\frac{m_i}{M}, \frac{\vec{\kappa}_i}{M} = O(\zeta)$. With similar restrictions on the electro-magnetic field one gets that the energy-momentum tensor of matter is $T^{AB} = T_{(1)}^{AB} + O(\zeta^2)$. This approximation is not reliable at distances from the point particles less than the gravitational radius $R_M = \frac{MG}{c^2} \approx 10^{25} M$ determined by the cutoff mass. The weak ADM Poincaré generators become equal to the Poincaré generators of this matter in inertial frames of Minkowski space-time plus terms of order $O(\zeta^2)$ containing GW's and matter. Finally the GW's described by this linearization must have wavelengths satisfying $\lambda/2\pi \approx L \gg R_M$. If all the particles are contained in a compact set of radius l_c (the source), we must have $l_c \gg R_M$ for particles with relativistic velocities and $l_c \geq R_M$ for slow particles (like in binaries).

With this Hamiltonian linearization we can avoid to make PN expansions, namely we get fully relativistic expressions, i.e. a PM Hamiltonian gravity.

In the second paper of Refs.[24] we have found the solutions of the super-momentum and super-Hamiltonian constraints and of the equations for the lapse and shift functions with the

Bianchi identities satisfied. Therefore we know the first order quantities $\pi_{(1)r}^{(\theta)}$, $\tilde{\phi} = 1 + 6\phi_{(1)}$, $1 + n_{(1)}$, $\bar{n}_{(1)(a)}$ (the action-at-a-distance part of the gravitational interaction) with an explicit expression for the PM Newton and gravito-magnetic potentials. In absence of the electro-magnetic field they are (the terms in $\Gamma_a^{(1)} = \sum_{\bar{a}r} \gamma_{\bar{a}a} R_{\bar{a}}$ describe the contribution of GW's)

$$\begin{aligned}
\tilde{\phi}(\tau, \vec{\sigma}) &= 1 + 6\phi_{(1)}(\tau, \vec{\sigma}) = \\
&= 1 + \frac{3G}{c^3} \sum_i \eta_i \frac{\sqrt{m_i^2 c^2 + \vec{\kappa}_i^2(\tau)}}{|\vec{\sigma} - \vec{\eta}_i(\tau)|} - \frac{3}{8\pi} \int d^3\sigma_1 \frac{\sum_a \partial_{1a}^2 \Gamma_a^{(1)}(\tau, \vec{\sigma}_1)}{|\vec{\sigma} - \vec{\sigma}_1|}, \\
\epsilon^4 g_{\tau\tau}(\tau, \vec{\sigma}) &= 1 + 2n_{(1)}(\tau, \vec{\sigma}) = \\
&= 1 - 2\partial_\tau^3 \mathcal{K}_{(1)}(\tau, \vec{\sigma}) - \frac{2G}{c^3} \sum_i \eta_i \frac{\sqrt{m_i^2 c^2 + \vec{\kappa}_i^2(\tau)}}{|\vec{\sigma} - \vec{\eta}_i(\tau)|} \left(1 + \frac{\vec{\kappa}_i^2}{m_i^2 c^2 + \vec{\kappa}_i^2}\right), \\
-\epsilon^4 g_{\tau a}(\tau, \vec{\sigma}) &= \bar{n}_{(1)(a)}(\tau, \vec{\sigma}) = \partial_a^3 \mathcal{K}_{(1)}(\tau, \vec{\sigma}) - \frac{G}{c^3} \sum_i \frac{\eta_i}{|\vec{\sigma} - \vec{\eta}_i(\tau)|} \left(\frac{7}{2} \kappa_{ia}(\tau) + \right. \\
&\quad \left. - \frac{1}{2} \frac{(\sigma^a - \eta_i^a(\tau)) \vec{\kappa}_i(\tau) \cdot (\vec{\sigma} - \vec{\eta}_i(\tau))}{|\vec{\sigma} - \vec{\eta}_i(\tau)|^2} \right) - \\
&\quad - \int \frac{d^3\sigma_1}{4\pi |\vec{\sigma} - \vec{\sigma}_1|} \partial_{1a} \partial_\tau \left[2\Gamma_a^{(1)}(\tau, \vec{\sigma}_1) - \int d^3\sigma_2 \frac{\sum_c \partial_{2c}^2 \Gamma_c^{(1)}(\tau, \vec{\sigma}_2)}{8\pi |\vec{\sigma}_1 - \vec{\sigma}_2|} \right], \\
\sigma_{(1)(a)(b)}|_{a \neq b}(\tau, \vec{\sigma}) &= \frac{1}{2} \left(\partial_a \bar{n}_{(1)(b)} + \partial_b \bar{n}_{(1)(a)} \right)|_{a \neq b}(\tau, \vec{\sigma}). \tag{5.3}
\end{aligned}$$

Then we have shown that the tidal variables $R_{\bar{a}}$ satisfy a wave equation ¹⁶

$$\begin{aligned}
\partial_\tau^2 R_{\bar{a}}(\tau, \vec{\sigma}) &\stackrel{\circ}{=} \Delta R_{\bar{a}}(\tau, \vec{\sigma}) + \sum_a \gamma_{\bar{a}a} \left[\partial_\tau \partial_a \bar{n}_{(1)(a)} + \right. \\
&\quad \left. + \partial_a^2 n_{(1)} + 2\partial_a^2 \phi_{(1)} - 2\partial_a^2 \Gamma_a^{(1)} + \frac{8\pi G}{c^3} T_{(1)}^{aa} \right](\tau, \vec{\sigma}). \tag{5.4}
\end{aligned}$$

If we use Eqs.(5.3) this equation becomes $(\partial_\tau^2 - \Delta) \sum_{\bar{b}} M_{\bar{a}\bar{b}} R_{\bar{a}} =$ (*known functional of matter*) with the D'Alembertian associated to the asymptotic Minkowski 4-metric and with $M_{\bar{a}\bar{b}} = \delta_{\bar{a}\bar{b}} - \sum_a \gamma_{\bar{a}a} \frac{\partial_a^2}{\Delta} \left(2\gamma_{\bar{b}a} - \frac{1}{2} \sum_b \gamma_{\bar{b}b} \frac{\partial_b^2}{\Delta} \right)$. The spatial operator $M_{\bar{a}\bar{b}}$ is the main difference between the 3-orthogonal gauges and the harmonic ones in the description of GW's.

Therefore, by using a no-incoming radiation condition based on the asymptotic Minkowski light-cone, we get a (complicated but tractable due to Ref.[51]) description of gravitational waves in these non-harmonic gauges, which can be connected to generalized TT(transverse traceless) gauges, as *retarded functions of the matter*. The results, restricted to the Solar System, are compatible the ones of the harmonic gauges used in the BCRS frame of Ref.[13].

¹⁶ For the tidal momenta one gets $\frac{8\pi G}{c^3} \Pi_{\bar{a}} = \partial_\tau R_{\bar{a}} - \sum_a \gamma_{\bar{a}a} \partial_a \bar{n}_{(1)(a)} + O(\zeta^2)$.

By using a Hamiltonian PM multipolar expansion in terms of Dixon multipoles [52–55] of the matter energy-momentum tensor we get a *relativistic mass quadrupole emission formula*. Moreover, notwithstanding there is no gravitational self-energy due to the Grassmann regularization, the energy, 3-momentum and angular momentum balance equations in GW emission are verified by *using the conservation of the asymptotic ADM Poincaré generators* (the same happens with the asymptotic Larmor formula of the electro-magnetic case with Grassmann regularization as shown in Refs.[56–59]).

These GW’s propagate in non-Euclidean instantaneous 3-spaces Σ_τ differing from the inertial asymptotic Euclidean 3-spaces at the first order (their intrinsic 3-curvature is determined by the GW’s and by the matter) and dynamically determined by the linearized solution of Einstein equations. These 3-spaces have a first order extrinsic curvature (with ${}^3K_{(1)}(\tau, \sigma^r) \approx F_{(1)}(\tau, \sigma^r)$ describing the clock synchronization convention, i.e. their shape as 3-sub-manifolds of space-time) and a first order modification of Minkowski light-cone.

In the third paper of Refs.[24] we eliminate the electro-magnetic field and we evaluate all the properties of these PM space-times: a) the 3-volume element, the 3-distance and the intrinsic and extrinsic 3-curvature tensors of the 3-spaces; b) the proper time of a time-like observer; c) the time-like and null 4-geodesics (they are relevant for the definition of the radial velocity of stars as shown in the IAU conventions of Ref.[60] and in study of *time delays*); d) the redshift and luminosity distance. In particular we find that the recessional velocity of a star is proportional to its luminosity distance from the Earth at least for small distances. This is in accord with the Hubble old redshift-distance relation which is formalized in the Hubble law (velocity-distance relation) when the standard cosmological model is used (see for instance Ref.[61] on these topics). These results have a dependence on the non-local York time, which could play a role in giving a different interpretation of the data from super-novae, which are used as a support for dark energy [45].

With the exception of the extrinsic 3-curvature tensor all the other quantities do not depend on the York time ${}^3K_{(1)}$ but on *non-local York time* (Δ is the Laplacian associated to the asymptotic Minkowski 4-metric)

$${}^3\mathcal{K}_{(1)}(\tau, \sigma^r) = \left(\frac{1}{\Delta} {}^3K_{(1)} \right)(\tau, \sigma^r). \quad (5.5)$$

In Subsection IIIB of the second paper in Refs.[24] it is shown that this HPM linearization can be interpreted as the first term of a Hamiltonian PM expansion in powers of the Newton constant G in the family of 3-orthogonal gauges. This expansion has still to be studied.

VI. THE POST-NEWTONIAN EXPANSION OF POST-MINKOWSKIAN HAMILTON EQUATIONS FOR THE PARTICLES AND DARK MATTER AS A RELATIVISTIC INERTIAL EFFECT DUE TO THE YORK TIME

We can write explicitly the linearized PM Hamilton equations for the particles and for the electro-magnetic field: among the forces there are both the inertial potentials and the GW’s.

In the third paper of Ref.[24] we disregarded electro-magnetism and we studied in more detail the PM equations of motion of the particles. If we use Eqs.(5.3) and the retarded solution of Eqs.(5.4) for the GW’s, the regularized equations of motion depend only on the

particles and have the form $\ddot{\vec{\eta}}_i(\tau) = \frac{1}{m_i} \vec{F}_i(\tau)$ with the forces depending on the positions and velocities of all the particles. Eqs.(5.3) imply that the equation for particle "i" is independent from m_i (equality of inertial and gravitational masses). The effective force $\vec{F}_i(\tau)$ contains

- a) the contribution of the lapse function $\tilde{n}_{(1)}$, which generalizes the Newton force;
- b) the contribution of the shift functions $\tilde{n}_{(1)(r)}$, which gives the gravito-magnetic effects;
- c) the retarded contribution of GW's, described by the functions $\Gamma_r^{(1)}$;
- d) the contribution of the volume element $\phi_{(1)}$ ($\tilde{\phi} = 1 + 6\phi_{(1)} + O(\zeta^2)$), always summed to the GW's, giving forces of Newton type;
- e) the contribution of the inertial gauge variable (the non-local York time) ${}^3\mathcal{K}_{(1)} = \frac{1}{\Delta} {}^3K_{(1)}$.

While in the electro-magnetic case in SR [49] the regularized equations of motion of the particles obtained by using the Lienard-Wiechert solutions for the electro-magnetic field are independent by the type of Green function (retarded or advanced or symmetric) used, this is not strictly true in the gravitational case. The effect of retardation is only pushed to $O(\zeta^2)$ and should appear in a study of the second order equations of motion.

Then we studied the Post-Newtonian (PN) expansion of these regularized PM equations of motion for the particles. We found that the particle 3-coordinates $\eta_i^r(\tau = ct) = \tilde{\eta}_i^r(t)$ satisfy the equation of motion

$$\begin{aligned} m_i \frac{d^2 \tilde{\eta}_i^r(t)}{dt^2} = & m_i \left[-G \frac{\partial}{\partial \tilde{\eta}_i^r} \sum_{j \neq i} \frac{m_j}{|\vec{\tilde{\eta}}_i(t) - \vec{\tilde{\eta}}_j(t)|} - \frac{1}{c} \frac{d\tilde{\eta}_i^r(t)}{dt} \left(\partial_t^2 |_{\vec{\tilde{\eta}}_i} {}^3\tilde{\mathcal{K}}_{(1)} + \right. \right. \\ & + 2 \sum_u v_i^u(t) \frac{\partial \partial_t |_{\vec{\tilde{\eta}}_i} {}^3\tilde{\mathcal{K}}_{(1)}}{\partial \tilde{\eta}_i^u} + \sum_{uv} v_i^u(t) v_i^v(t) \frac{\partial^2 {}^3\tilde{\mathcal{K}}_{(1)}}{\partial \tilde{\eta}_i^u \partial \tilde{\eta}_i^v} \left. \right) (t, \vec{\tilde{\eta}}_i(t)) \Big] + \\ & + F_{(1PN)}^r(t) + (\text{higher PN orders}), \end{aligned} \quad (6.1)$$

where at the lowest order we find the standard Newton gravitational force $\vec{F}_{i(Newton)}(t) = -m_i G \frac{\partial}{\partial \tilde{\eta}_i^r} \sum_{j \neq i} \frac{m_j}{|\vec{\tilde{\eta}}_i(t) - \vec{\tilde{\eta}}_j(t)|}$.

If we put ${}^3K_{(1)} = 0$, the forces $\vec{F}_{Newton} + \vec{F}_{(1PN)}$ reproduce the standard results about binaries found with a different type of approximation in Ref.[62].

Therefore the (arbitrary in these gauges) double rate of change in time of the trace of the extrinsic curvature creates a 0.5 PN damping (or anti-damping since the sign of the inertial gauge variable ${}^3\mathcal{K}_{(1)}$ of Eq.(5.5) is not fixed) effect on the motion of particles. This is an *inertial effect* (hidden in the lapse function) not existing in Newton theory where the Euclidean 3-space is absolute.

In the 2-body case we get that for Keplerian circular orbits of radius r the modulus of the relative 3-velocity can be written in the form $\sqrt{\frac{G(m+\Delta m(r))}{r}}$ with $\Delta m(r)$ function only of ${}^3\mathcal{K}_{(1)}$.

Now the *rotation curves of spiral galaxies* (see Refs.[63–65] for reviews) imply that the relative 3-velocity goes to constant for large r (instead of vanishing like in Kepler theory). This result can be simulated by fitting $\Delta m(r)$ (i.e. the non-local York time) to the experimental

data: as a consequence $\Delta m(r)$ is interpreted as a *dark matter halo* around the galaxy. With our approach this dark matter would be a *relativistic inertial effect* consequence of the a non-trivial shape of the non-Euclidean 3-space as a 3-sub-manifold of space-time.

We find that Eq.(6.1) can be rewritten in the form

$$\begin{aligned} \frac{d}{dt} \left[m_i \left(1 + \frac{1}{c} \frac{d}{dt} {}^3\tilde{\mathcal{K}}_{(1)}(t, \vec{\eta}_i(t)) \right) \frac{d\vec{\eta}_i^r(t)}{dt} \right] \stackrel{\circ}{=} -G \frac{\partial}{\partial \vec{\eta}_i^r} \sum_{j \neq i} \eta_j \frac{m_i m_j}{|\vec{\eta}_i(t) - \vec{\eta}_j(t)|} + \\ + \mathcal{O}(\zeta^2). \end{aligned} \quad (6.2)$$

We see that the term in the non-local York time can be *interpreted* as the introduction of an *effective (time-, velocity- and position-dependent) inertial mass term* for the kinetic energy of each particle: $m_i \mapsto m_i \left(1 + \frac{1}{c} \frac{d}{dt} {}^3\tilde{\mathcal{K}}_{(1)}(t, \vec{\eta}_i(t)) \right)$ in each instantaneous 3-space. Instead in the Newton potential there are the gravitational masses of the particles, equal to the inertial ones in the 4-dimensional space-time due to the equivalence principle. Therefore the effect is due to a modification of the effective inertial mass in each non-Euclidean 3-space depending on its shape as a 3-sub-manifold of space-time: *it is the equality of the inertial and gravitational masses of Newtonian gravity to be violated!* In Galilei space-time the Euclidean 3-space is an absolute time-independent notion like Newtonian time: the non-relativistic non-inertial frames live in this absolute 3-space differently from what happens in SR and GR, where they are (in general non-Euclidean) 3-sub-manifolds of the space-time.

A similar interpretation can be given for the other two main signatures of the existence of dark matter in the observed masses of galaxies and clusters of galaxies, namely the virial theorem [66–68] and weak gravitational lensing [69] [67, 70].

This option for explaining (at least part of) dark matter differs:

- 1) from the non-relativistic MOND approach [71] (where one modifies Newton equations);
- 2) from modified gravity theories like the $f(R)$ ones (see for instance Refs.[72]; here one gets a modification of the Newton potential);
- 3) from postulating the existence of WIMP particles [73].

Let us also remark that the 0.5PN effect has origin in the lapse function and not in the shift one, as in the gravito-magnetic elimination of dark matter proposed in Ref.[74].

As a consequence of the property of non-Euclidean 3-spaces of being 3-sub-manifolds of Einstein space-times (independently from cosmological assumptions) there is the possibility of describing part (or maybe all) dark matter as a *relativistic inertial effect*. As we have seen the three main experimental signatures of dark matter can be explained in terms of the non-local York time ${}^3\mathcal{K}_{(1)}(\tau, \vec{\sigma})$, the inertial gauge variable describing the general relativistic remnant of the gauge freedom in clock synchronization.

VII. CLOCK SYNCHRONIZATION AND RELATIVISTIC METROLOGY

Since in GR the gauge freedom is the arbitrariness in the choice of the 4-coordinates, a similar arbitrariness is expected in the non-inertial frames of SR and this is described in Refs. [21, 22].

However, at the experimental level the description of matter (and also of the spectra of light from stars) is not based on DO's or 4-scalars but is *intrinsically coordinate-dependent*, namely is connected with the *metrological conventions* used by physicists, engineers and astronomers for the modeling of space-time (see Ref.[75] for a review on relativistic metrology). The basic conventions are

- a) An atomic clock as a standard of time;
- b) The 2-way velocity of light in place of a standard of length;
- c) A conventional reference frame centered on a given observer as a standard of space-time (GPS is an example of such a standard);

and the adopted astronomical reference frames are:

A) The description of satellites around the Earth is done by means of NASA coordinates [41] either in ITRS (the terrestrial frame fixed on the Earth surface)[42] or in GCRS (the geocentric frame centered on the Earth center) (see Ref.[43]).

B) The description of planets and other objects in the Solar System uses BCRS (a barycenter quasi-inertial Minkowski frame, if perturbations from the Milky Way are ignored ¹⁷), centered in the barycenter of the Solar System, and ephemerides (see Ref.[43]).

C) In astronomy the positions of stars and galaxies are determined from the data (luminosity, light spectrum, angles) on the sky as living in a 4-dimensional nearly-Galilei space-time with the celestial ICRS [45] frame considered as a "quasi-inertial frame" (all galactic dynamics is Newtonian gravity), in accord with the assumed validity of the cosmological and Copernican principles. Namely one assumes a homogeneous and isotropic cosmological Friedmann-Robertson - Walker solution of Einstein equations (the standard Λ CDM cosmological model). In it the constant intrinsic 3-curvature of instantaneous 3-spaces is nearly zero as implied by the CMB data [45], so that Euclidean 3-spaces (and Newtonian gravity) can be used. However, to reconcile all the data with this 4-dimensional reconstruction one must postulate the existence of dark matter and dark energy as the dominant components of the classical universe after the recombination 3-surface!

What is still lacking is a PM extension of the celestial frame such that the PM BCRS frame is its restriction to the solar system inside our galaxy. Hopefully this will be achieved with the ESA GAIA mission devoted to the cartography of the Milky Way [77].

The recombination 3-surface is the natural Cauchy surface for using classical GR in the description of the 3-universe after the end of the preceding quantum phases of its evolution (it is a kind of Heisenberg cut between quantum cosmology and classical astrophysics). Let us also remark that the fixed stars of star catalogues [44] may be considered as a phenomenological definition of *spatial infinity* in asymptotically Minkowskian space-times:

¹⁷ Essentially it is defined as a *quasi-inertial system, non-rotating* with respect to some selected fixed stars, in Minkowski space-time with nearly-Euclidean Newton 3-spaces. The qualification *quasi-inertial* is introduced to take into account GR, where inertial frames exist only locally. More exactly it is a PM Einstein space-time with 3-spaces having a very small extrinsic curvature of order c^{-2} and with a PN treatment of the gravitational field of the Sun and of the planets in a special harmonic gauge of Einstein GR (see Ref.[76] for possible gravitational anomalies inside the Solar System).

their spatial axes define an asymptotic inertial frame centered on an asymptotic inertial observer.

Therefore, in every generally covariant theory of gravity the freedom in the choice of the gauge, i.e. of the 4-coordinate system of space-time and of the time-like observer origin of the 3-coordinates, disappears when we want to make comparison with experimental data: we must choose those mathematical gauges which are compatible with the metrological conventions.

Since, as said in the Introduction, at the experimental level *the description of matter is intrinsically coordinate-dependent*, namely is connected with the conventions used by physicists, engineers and astronomers for the modeling of space-time, we have to choose a gauge (i.e. a 4-coordinate system) in non-modified Einstein gravity which is in agreement with the observational conventions in astronomy.

Since ICRS [44] has diagonal 3-metric, our 3-orthogonal gauges are a good choice. We are left with the inertial gauge variable ${}^3\mathcal{K}_{(1)} = \frac{1}{\Delta} {}^3K_{(1)}$ not existing in Newtonian gravity. As already said the suggestion is to try to fix ${}^3\mathcal{K}_{(1)}$ in such a way to eliminate dark matter as much as possible.

The open problem is the determination of the non-local York time from the data. From what is known from the Solar System and from inside the Milky Way near the galactic plane, it seems that it is negligible near the stars inside a galaxy. On the other hand, it is non zero near galaxies and clusters of galaxies of big mass. However only a mean value in time of time- and space-derivatives of the non-local York time can be extracted from the data. At this stage it seems that the non-local York time is relevant around the galaxies and the clusters of galaxies where there are big concentrations of mass and the dark matter haloes and that it becomes negligible inside the galaxies where there is a lower concentration of mass. Instead there is no indication on its value in the voids existing among the clusters of galaxies.

However if we do not *know the non-local York time on all the 3-universe* at a given τ we cannot get an experimental determination of the York time ${}^3K_{(1)}(\tau, \vec{\sigma}) = \Delta {}^3\mathcal{K}_{(1)}(\tau, \vec{\sigma})$. Therefore some phenomenological parametrizations of ${}^3\mathcal{K}_{(1)}(\tau, \vec{\sigma})$ will have to be devised to see the implications for ${}^3K_{(1)}(\tau, \vec{\sigma})$. As said in the Introduction, a phenomenological determination of the York time would help in trying to get a PM extension of the existing quasi-inertial Galilei Celestial reference frame (ICRS). Then automatically BCRS would be its quasi-Minkowskian approximation for the Solar System. Let us remark that the 3-spaces can be quasi-Euclidean (i.e. with a small 3-curvature tensor), as required by CMB data [45] in the astrophysical context, even when their shape as 3-sub-manifolds of space-time is not trivial and is described by a not-small York time.

This would be the way out from the gauge problem in general relativity: the observational conventions for matter would select a reference system of 4-coordinates for PM space-times in the associated 3-orthogonal gauge.

VIII. CONCLUSIONS

I have given a full review of an approach to asymptotically Minkowskian classical canonical GR based on a description of global non-inertial frames centered on a time-like observer

which is suggested by relativistic metrology. The gauge freedom in clock synchronization, which does not exist in Galilei space-time (Newton time is absolute) and is not restricted in Minkowski space-time (the whole class of admissible 3+1 splittings in this absolute space-time is allowed), is restricted to the gauge freedom connected with the inertial gauge variable 3K , the York time, which determines the shape of the instantaneous (in general non-Euclidean) 3-spaces as 3-sub-manifolds of the space-time.

The study of canonical ADM tetrad gravity in asymptotically Minkowskian space-times in the York canonical basis allowed to find the family of non-harmonic 3-orthogonal Schwinger time gauges and to define a HPM linearization in them. The main properties of these non-harmonic gauges are that only the GW's (but not the lapse and shift functions) are retarded quantities with a no-incoming radiation condition and that one can naturally find which quantities depend upon the York time.

I have described relativistic particle mechanics in GR. The more surprising result is that in the PN expansion of the PM equations of motion there is a 0.5PN term in the forces depending upon the York time. This opens the possibility to describe dark matter as a relativistic inertial effect implying that the effective inertial mass of particles in the 3-spaces is bigger of the gravitational mass because it depends on the York time (i.e. on the shape of the 3-space as a 3-sub-manifold of the space-time: this is impossible in Newton gravity in Galilei space-time and leads to a violation of the Newtonian equivalence principle).

At a more mathematical level some open problems under investigation are:

A) The quantization of the massive Klein-Gordon field in non-inertial frames of Minkowski space-time. Is it possible to evade the no-go theorem of Ref.[78] and to get unitary evolution? And to extend to GR?

B) Find the second order of the HPM expansion to see whether in PM space-times there is the emergence of hereditary terms (see Refs.[50, 79]) like the ones present in harmonic gauges. Like in standard approaches (see the review in Appendix A of the second paper in Refs.[24]) regularization problems may arise at the higher orders.

C) Study the PM equations of motion of the transverse electro-magnetic field trying to find Lienard-Wiechert-type solutions (see Subsection VB of the second paper in Refs.[24]). Study astrophysical problems where the electro-magnetic field is relevant.

D) Try to find the final true DO's of GR by using approximate solutions of the super-Hamiltonian and super-momentum constraints. This would allow to make a multi-temporal quantization (see Ref. [80]) of the approximate solution (for instance the linearized HPM theory over the asymptotic Minkowski space-time as shown in the second paper of Ref.[39]), in which only the tidal DO variables are quantized but not the inertial gauge ones. In this way the space-time would remain a 4-manifold with the gauges determined by relativistic metrology: only the eigenvalues of the 3-metric of the non-Euclidean 3-spaces would be quantized with an induced quantization of 3-lengths, 3-areas and 3-volumes to be compared with the results of loop quantum gravity.

E) Find the canonical transformation from the York canonical basis to the Ashtekar variables [81, 82], [4, 12], in asymptotically Minkowskian space-times.

Instead at the physical level the next big challenge after dark matter is *dark energy* in cosmology [45] (see Ref.[83, 84] for what we really know). Even if in cosmology we cannot use

canonical gravity, in the first paper of Ref.[24] it is shown that the usual non-Hamiltonian 12 ADM equations can be put in a form allowing to use the interpretations based on the York canonical basis by means of the expansion and the shear of the Eulerian observers.

Let us remark that in the Friedmann-Robertson-Walker (FRW) cosmological solution the Killing symmetries connected with homogeneity and isotropy imply (τ is the cosmic time, $a(\tau)$ the scale factor) ${}^3K(\tau) = -\frac{\dot{a}(\tau)}{a(\tau)} = -H$, namely the York time is no more a gauge variable but coincides with the Hubble constant. However at the first order in cosmological perturbations we have ${}^3K = -H + {}^3K_{(1)}$ with ${}^3K_{(1)}$ being again an inertial gauge variable. Instead in inhomogeneous space-times without Killing symmetries like the Szekeres ones [85–87] the York time remains an inertial gauge variable.

Also in the back-reaction approach [88–90] (see also Ref.[91]) to dark energy, according to which dark energy is a byproduct of the non-linearities of general relativity when one considers spatial mean values on large scales to get a cosmological description of the universe taking into account the inhomogeneity of the observed universe, one gets that the spatial average of the York time (a 3-scalar gauge variable) gives the effective Hubble constant of that approach.

Therefore the York time has a central position also in the main quantities on which relies the interpretation of dark energy in the standard Λ CDM cosmological model (Hubble constant, the old Hubble redshift-distance relation replaced in FRW cosmology with the velocity distance relation or Hubble law). As a consequence it looks reasonable to investigate on a possible gauge origin also of dark matter.

As a first step we have considered a perfect fluid as matter in the first order of HPM expansion [92] adapting to tetrad gravity the special relativistic results of Refs.[93, 94] (based on the approach of Ref.[95]). Since in our formalism all the canonical variables in the York canonical basis, except the angles θ^i , are 3-scalars, we can complete Buchert’s formulation of back-reaction [88] by taking the spatial average of nearly all the PM Hamilton equations in our non-harmonic 3-orthogonal gauges. This will allow to make the transition from the PM space-time 4-metric to an inhomogeneous cosmological one (only conformally related to Minkowski space-time at spatial infinity) and to try to reinterpret also the dark energy as a relativistic inertial effect (and not only as a non-linear effect of inhomogeneities). The role of the York time, now considered as an inertial gauge variable, in the theory of back-reaction and in the identification of what is called dark energy ¹⁸ is completely unexplored.

Moreover the recent point of view of Ref.[96] taking into account the relevance of the voids among the clusters of galaxies suggests to try to develop a phenomenological parametrization of the York time to see whether we can simultaneously fit the data on dark matter and make contact with the back-reaction approach to dark energy.

Finally let us remark that in Eq.(4.5) we showed that in the York canonical basis the York time contributes with a negative term to the kinetic energy in the ADM energy. It would also play a role in a study to be done on the reformulation of the Landau-Lifschitz energy-momentum pseudo-tensor as the energy-momentum tensor of a viscous pseudo-fluid. It could be possible that for certain choices of the York time the resulting effective equation of state has negative pressure, realizing also in this way a simulation of dark energy.

¹⁸ As we have already said at the PM level the red-shift and the luminosity distance depend upon the York time, and this could play a role in the interpretation of the data from super-novae.

Is it possible to find a 3-orthogonal gauge in a inhomogeneous Einstein space-time in which the inertial gauge variable York time allows to eliminate both dark matter and dark energy through the choice of a 4-coordinate system to be used as a consistent PM ICRS saving the main good properties of the standard Λ CDM model?

As a final remark let us notice that when there is a perfect fluid with unit time-like 4-velocity $U^A(\tau, \vec{\sigma})$, there is also the congruence of its time-like flux curves: in general it is not surface-forming and it is independent from the previous two congruences. If $(U^A(\tau, \vec{\sigma}); \mathcal{U}_{(a)}^A(\tau, \vec{\sigma}))$ is an ortho-normal tetrad carried by a flux line, the connection of these 4-vectors to the ortho-normal tetrad of the Eulerian observers is

$$\begin{aligned} U^A(\tau, \vec{\sigma}) &= \Gamma \left(l^A + \sum_a \beta_{(a)} {}^4\bar{E}_{(a)}^{\circ A} \right) (\tau, \vec{\sigma}), \\ \mathcal{U}_{(a)}^A(\tau, \vec{\sigma}) &= \left(t_{(a)} l^A + \sum_b \gamma_{(a)(b)} {}^4\bar{E}_{(b)}^{\circ A} \right) (\tau, \vec{\sigma}), \end{aligned} \quad (8.1)$$

with $t_{(a)}(\tau, \vec{\sigma}) = (\sum_b \gamma_{(a)(b)} \beta_{(b)}) (\tau, \vec{\sigma})$ and $[\sum_{cd} (\delta_{(c)(d)} - \beta_{(c)} \beta_{(d)}) \gamma_{(a)(c)} \gamma_{(b)(d)}] (\tau, \vec{\sigma}) = \delta_{(a)(b)}$. When the vorticity of the fluid vanishes, so that its 4-velocity is surface forming, there is a 3+1 splitting of space-time determined by the irrotational fluid. While in SR we can always choose a global non-inertial frame coinciding with these 3+1 splitting, in GR we have to show that there is a gauge fixing on the inertial gauge variable ${}^3K(\tau, \vec{\sigma})$ (the York time) allowing this identification. These problems are studied in Ref.[92].

Let us remark that Eqs.(8.1) establish the bridge between our 3+1 point of view and the 1+3 point of view of Refs.[97–99], where one describes both the gravitational field and the matter as seen by a generic family of observers with 4-velocity $U^A(\tau, \vec{\sigma})$. Most of the results in cosmology (see for instance Refs.[100, 101]) are presented in the 1+3 framework. However, in the 1+3 point of view vorticity is an obstruction to formulate the Cauchy problem (3-spaces are not existing; each observer uses as rest frame the tangent 3-space orthogonal to the 4-velocity) and there is no natural way to identify the inertial gravitational gauge variables of the Hamiltonian formalism based on Dirac’s constraint theory (see also Appendix A of the first paper in Refs.[24]).

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